A NEW FORMULATION FOR THE ANALYSIS OF NON-UNIFORM TRANSMISSION LINES USING FREQUENCY-VARYING BASIS FUNCTIONS

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Abstract - A new method of moment based formulation for the solution of the telegraphist's equations in non-uniform transmission lines is presented. Entire domain basis function that build-in a frequency variation are used to cover wider frequency and physical dimension ranges. The results obtained using the proposed formulation are validated by comparison to those obtained by a CAD package and to measured data.

Introduction

Today's high performing MICs must meet stringent electrical performance requirement while being low-cost and occupying the minimum amount of space. In this context, non-uniform circuit transitions are often required either by constraints or by design. To utilize the full potential of such circuits, however, fast and accurate design tools must be developed. Unfortunately, aside from costly fully three-dimensional field simulators, and with the exception of the numerical approach described in [1-2] or the analytical solution for single-line exponential tapers (see for example [3]), little is available in the way of practical modeling and simulation techniques of these circuits.

In this paper, a new formulation of the non-uniform transmission line problem is presented. The proposed approach is applied to the analysis of a number of non-uniform microstrip transitions. The accuracy of the proposed approach is validated by comparison with a CAD package results and with measured data.

II. Formulation

A schematic representation of a non-uniform section of a microstrip line is shown in Figure 1. For the fundamental microstrip mode, the propagation in such a structure can be described in terms of the telegraphers' equations with frequency and position dependent line parameters, namely:

$$\begin{cases} \dfrac{\partial I(f,z)}{\partial z} = -Y(f,z)V(z) \\ \dfrac{\partial V(f,z)}{\partial z} = -Z(f,z)I(z) \end{cases} \tag{1}$$

where f is the frequency and Z and Y are the per-unit length impedance and admittance of the line, respectively. These parameters are assumed to be known from the line geometry at a given z-position and from the frequency. Their computation can be carried out by a number of different numerical techniques with varying degrees of accuracy and computational cost. However, for the purpose of having a practical design tool, the accurate closed form expressions in [4] are used to minimize the computational resources required.

Next, we proceed to formulate a method of moment solution of the coupled equations in (1). The key to such a solution is the accurate representation of the unknown current and voltage along the line. First, we note that a conventional sub-domain basis function expansion approach would not work here. This is due to the discontinuity that would result in either the current or voltage as a consequence of the derivative with respect to z and the coupling of equations (1). Therefore, an entire-domain basis function formulation is needed. In [1-2], one such approach was used where all quantities (I, V, Z, Y) were expanded in terms of a set of series of Chebychev polynomials. After some algebra, a matrix equation for the unknown current and voltage coefficients was obtained which would yield the S-parameters of structure investigated upon solution. This approach will work rather well provided that the electric length of the line is such that the number of minima and maxima in the current/voltage distributions does not exceed N, the total number of Chebychev

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polynomials used in their expansion. Consequently, a large number of polynomials is generally required to insure accuracy over a wide range of frequencies and physical dimensions.

In the present approach, we use frequency-varying basis functions by expanding the currents and voltages in terms of forward and backward propagating waves with different wavenumbers such that:

$$\begin{cases} I(z)=\sum_{i=1}^{N} a_i e^{-j\beta_i z}+b_i e^{j\beta_i z}=\sum_{i=1}^{N} a_i F_i(z)+b_i B_i(z) \\ V(z)=\sum_{i=1}^{N} c_i e^{-j\beta_i z}+d_i e^{j\beta_i z}=\sum_{i=1}^{N} c_i F_i(z)+d_i B_i(z) \end{cases} \quad (2)$$

where $\{a_i, b_i, c_i, d_i\}$ are unknown coefficients and where the frequency-dependence is built in the propagation constants β_i. Note that the expansion in (2) is not a Fourier representation since the set of wavenumbers $\{\beta_i\}$ is not related to the spatial coordinate z and the line length L, but rather to the line's cross-sectional dimensions at a set of points $\{z_i\}$ along the line and to frequency. Substituting (2) into (1) and testing with F_n and B_n, we obtain a matrix equation whose entries are given in terms of the following inner products:

$$\langle F_i, F_n\rangle \langle F_i, B_n\rangle \langle B_i, F_n\rangle \langle B_i, B_n\rangle \quad (3.a)$$

$$\langle Z(z)F_i, F_n\rangle \langle Z(z)F_i, B_n\rangle \langle Z(z)B_i, F_n\rangle \langle Z(z)B_i, B_n\rangle \quad (3.b)$$

$$\langle Y(z)F_i, F_n\rangle \langle Y(z)F_i, B_n\rangle \langle Y(z)B_i, F_n\rangle \langle Y(z)B_i, B_n\rangle \quad (3.c)$$

where the inner product definition used is:

$$\langle f,g\rangle=\int_0^L f(z)g(z)dz \quad (4)$$

The terms of equation (3.a) are easily evaluated in closed form. However, to evaluate the terms of equations (3.b) and (3.c), a slightly different procedure is followed. First, the total line length is subdivided in to M equal segments (see Figure 1). Then, the per-unit length parameters $Y(f, z)$ and $Z(f, z)$ are represented by a piece-wise linear function such that, at the given frequency f and over the m^{th} segment we have:

$$\begin{cases} Y(z)=A_{ym}z+B_{ym} \\ Z(z)=A_{zm}z+B_{zm} \end{cases} \quad \text{for } z_m \le z \le z_{m+1} \quad (5)$$

where A_{ym}, A_{zm}, B_{ym} and B_{zm} are computed from $Y(z_m)$, $Y(z_{m+1})$, $Z(z_m)$ and $Z(z_{m+1})$. The integrals of (3.b) and (3.c) can then be written as sums of the general form:

$$\langle X(z)f,g\rangle=\sum_{m=1}^{M} A_{xm}\int_{z_m}^{z_{m+1}} z\cdot f(z)g(z)dz+B_{xm}\int_{z_m}^{z_{m+1}} f(z)g(z)dz \quad (6)$$

where $f(z)$ and $g(z)$ represent combinations of the functions F_i and B_i. Consequently, closed form expressions for these integrals are easily obtained in terms of the known $A_{.m}$ and $B_{.m}$ coefficients at each frequency.

With the method of moments matrix thus filled, boundary conditions are applied to complete the system of equations and compute the scattering parameters of the non-uniform line. This is done by considering the terminal conditions shown in Figure 1 which give:

$$\begin{cases} V(0)=V_{01}-Z_{01}I(0) \\ V(L)=V_{02}+Z_{02}I(L) \end{cases} \quad (7)$$

where $Z_{01} = Z_{02} = 50\ \Omega$. To obtain the scattering parameters of the line, we solve the problem twice: once with $\begin{bmatrix} V_{01}=1 \\ V_{02}=0 \end{bmatrix}$ and a second time with $\begin{bmatrix} V_{01}=0 \\ V_{02}=1 \end{bmatrix}$. Using these results, and the S-parameters definition: $S_{ij}=\frac{2V_{ij}-E_{ij}}{E_{it}}$ where V_{ij} is the voltage at port i when port j is excited and V_{0i} is the excitation voltage at port i ($V_{0j} = 0$ for $i \neq j$), we obtain the four S-parameters of the line.

III. Results

The above approach has been implemented and tested on a number of structures with good results. With only a moderate number of basis functions (between 3 and 5), a wide range of linear as well as arbitrary microstrip tapers were investigated and compared to MDS models [5] with excellent agreement. Further validation was carried out by analyzing width-modulated lines and comparing to MDS, to measurements and to published results. Figure 2 shows the scattering parameters of an end-to-end taper simulated in MDS by cascading two linear tapers and in the present approach by describing the line width, w, by the function $w(z)=w1+\Delta w\left(1-\frac{|z-L/2|}{L/2}\right)$. The agreement obtained is excellent over the entire frequency range of the simulation. Figure 3 shows the resulting S-parameters when the end-to-end tapers of Figure 2 are used as the unit cell for a periodic structure. The results for six periods are again compared to those obtained using cascaded tapers in MDS. Good agreement is again seen and the expected filter-like behavior of this structure is shown. Figure 4 shows the computed and measured S-parameters for a sinusoidally-modulated periodic microstrip structure where the strip width is given by the function $w(z)=w_0(1-m\cos(\frac{2\pi z}{L}))$. Good agreement between the simulated and measured results is once again seen. These results compare well also with those obtained by [6] using a cascading approach.

IV. Conclusion

A new formulation using a method of moments approach with frequency-varying basis functions for the simulation of non-uniform transmission lines has been presented. The accuracy of the proposed technique was tested by comparison to existing empirical models and to measured data. The fact that basis functions chosen build in the frequency dependence of the current and voltage makes it possible to solve complicated structures with only a few basis functions giving this method a good efficiency. With such an efficient technique, circuit synthesis problems using non-uniform lines for a variety of application are now undertaken.

References

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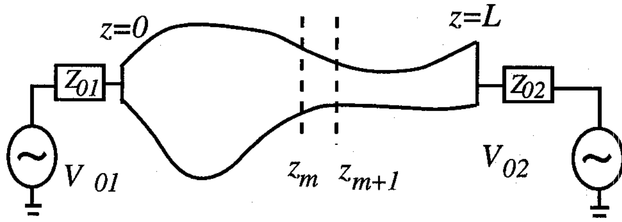
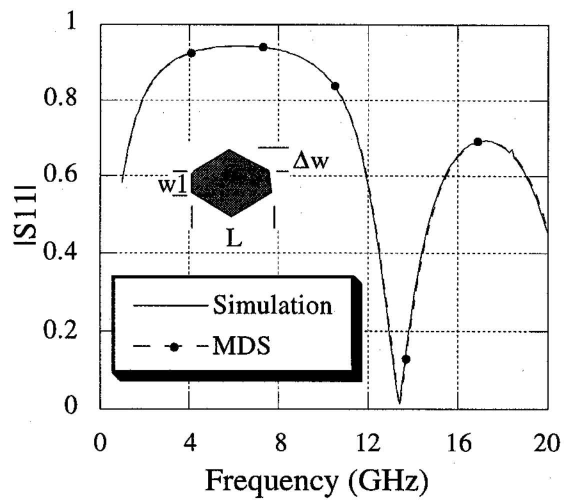


Figure 1. Schematic representation of the geometry of a non-uniform line.



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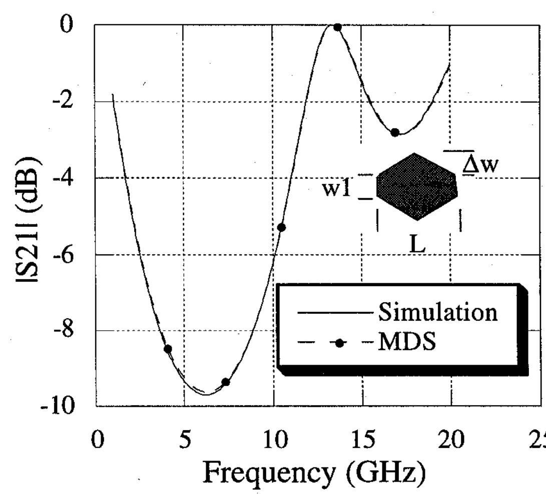

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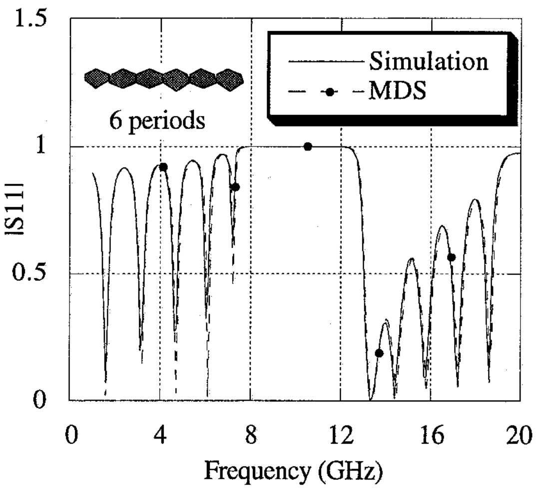
Figure 2. Scattering parameters of an end-to-end taper. w1=0.75mm, Δw=3.5mm, L=5mm, ε_r=10, h=0.254mm.



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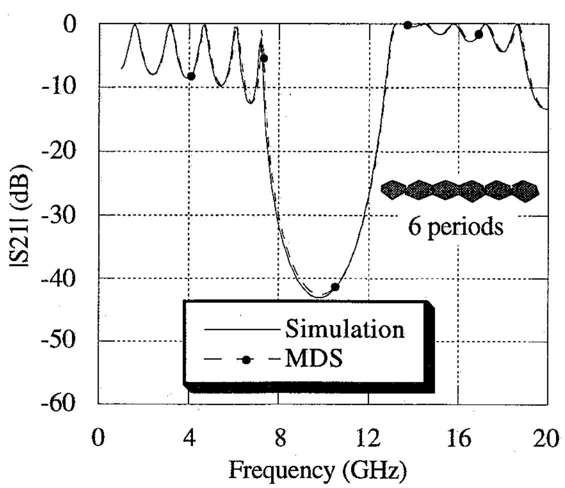

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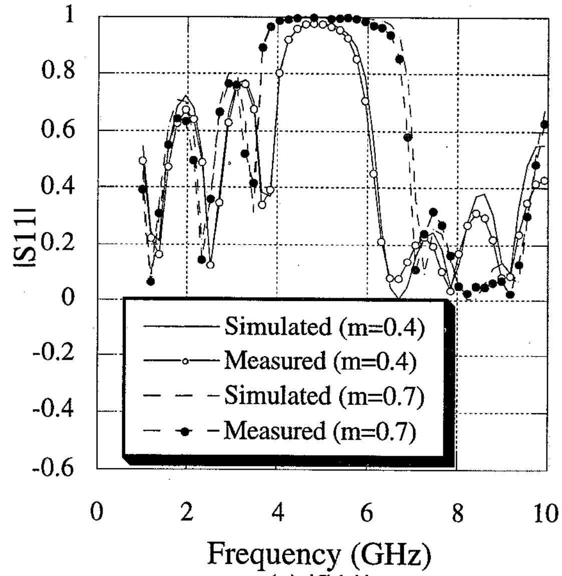
Figure 3. Scattering parameters of a periodic structure with the unit cell of figure 2.



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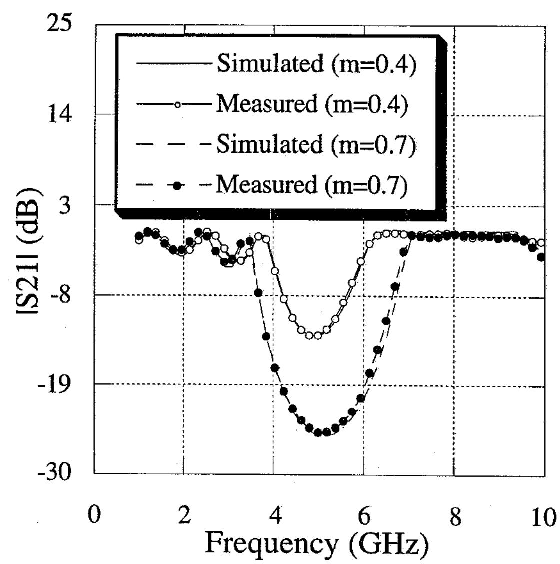

(b) |S21|

Figure 4. Computed and measured scattering parameters of a sinusoildally width-modulated microstrip line. w0=0.25cm, number of periods=4, L=4.0cm, ε_r=10.2, h=0.635mm.